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Membranes on an Orbifold

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Abstract

We harvest clues to aid with the interpretation of the recently discovered $\mathcal{N} = 8$ supersymmetric Chern-Simons theory with $SO(4)$ gauge symmetry. The theory is argued to describe two membranes moving in the orbifold $\mathbf{R}^8/\mathbf{Z}_2$. At level $k = 1$ and $k = 2$, the classical moduli space \mathcal{M} coincides with the infrared moduli space of $SO(4)$ and $SO(5)$ super Yang-Mills theory respectively. For higher Chern-Simons level, the moduli space is a quotient of \mathcal{M} . At a generic point in the moduli space, the massive spectrum is proportional to the area of the triangle formed by the two membranes and the orbifold fixed point.

Introduction

In [1], a novel, conformally invariant, Lagrangian in $d = 2 + 1$ dimensions was constructed. The theory enjoys maximal supersymmetry and a manifest $SO(8)$ R-symmetry, strongly suggesting that it describes the low-energy dynamics of multiple M2-branes in M-theory. Various aspects of this theory were anticipated in [2, 3, 4] and a number of recent papers have explored some of its properties [5, 6, 7, 9, 8]. Yet so far the interpretation in terms of M2-branes has remained somewhat murky. The purpose of this short note is to shed some light on this issue through a study of the classical vacuum moduli space and spectrum of the theory.

We work with the simplest – and, to date, only – explicit example of the Lagrangian, which is based on an $SO(4)$ gauge symmetry with an integer valued coupling constant k . We will show that, at levels $k = 1$ and $k = 2$, the classical moduli space \mathcal{M} coincides with the infra-red limit of $SO(4)$ and $SO(5)$ super Yang-Mills theory. This describes two membranes moving in the background of the orbifold $\mathbf{R}^8/\mathbf{Z}_2$, without and with discrete torsion respectively. For $k > 2$, we find that the vacuum moduli space is the quotient of \mathcal{M} . The group acts on the moduli space, but does not appear to have a natural action on the underlying spacetime. We further show that, at a generic point in the moduli space, the mass of the heavy states is proportional to the area of the triangle formed by the two membranes and the fixed point, and make some comments on the implications of this mass formula.

The M2-Brane Lagrangian

The Lagrangian presented in [1] is built around a 3-algebra \mathcal{A} . This is a vector space with basis T^a , $a = 1, \dots, \dim \mathcal{A}$, endowed with a trilinear antisymmetric product,

$$[T^a, T^b, T^c] = f^{abc}_d T^d. \quad (1)$$

The algebra is accompanied by an inner product, $h^{ab} = \text{Tr}(T^a T^b)$, with which indices may be raised and lowered. The structure constants of the algebra are then required to be totally anti-symmetric, $f^{abcd} = f^{[abcd]}$, and to satisfy the “fundamental identity”

$$f^{aef}_g f^{bcdg} - f^{bef}_g f^{acdg} + f^{cef}_g f^{abdg} - f^{def}_g f^{abcg} = 0. \quad (2)$$

The matter fields consist of 8 algebra-valued scalar fields X^I_a , $I = 1, \dots, 8$, transforming in the $\mathbf{8}_v$ of $SO(8)$, together with algebra-valued spinors Ψ^a transforming in the $\mathbf{8}_s$ of $SO(8)$. The theory also includes a non-propagating gauge field A^{ab}_μ . The dynamics is

governed by the Lagrangian,

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2}\mathcal{D}^\mu X^{Ia}\mathcal{D}_\mu X_a^I + \frac{i}{2}\bar{\Psi}^a\Gamma^\mu\mathcal{D}_\mu\Psi_a + \frac{i}{4}\bar{\Psi}_b\Gamma_{IJ}X_c^IX_d^J\Psi_af^{abcd} \\ & -V(X) + \frac{1}{2}\epsilon^{\mu\nu\lambda}\left(f_{abcd}A_\mu^{ab}\partial_\nu A_\lambda^{cd} + \frac{2}{3}f_{cda}{}^gf_{efgb}A_\mu^{ab}A_\nu^{cd}A_\lambda^{ef}\right),\end{aligned}\quad (3)$$

where the scalar potential is

$$V(X) = \frac{1}{12}f_{abcd}f_{efgd}X^{Ia}X^{Jb}X^{Kc}X^{Ie}X^{Jf}X^{Kg},\quad (4)$$

while the covariant derivative is defined by

$$\mathcal{D}_\mu X^{Ia} = \partial_\mu X^{Ia} + f_{bcd}^a A_\mu^{cd} X^{Ib}.\quad (5)$$

The theory is invariant under 16 supercharges and the gauge symmetry:

$$\begin{aligned}\delta X_a^I &= -f_{abcd}\Lambda^{bc}X^{Id} \\ \delta\Psi^a &= -f^{abcd}\Lambda_{bc}\Psi^d \\ f_{abcd}\delta A_\mu^{ab} &= f_{abcd}\mathcal{D}_\mu\Lambda^{ab}.\end{aligned}\quad (6)$$

Presently, the only known, finite-dimensional, representation of a 3-algebra has $\dim \mathcal{A} = 4$ and the gauge field A_μ^{ab} is valued in $so(4)$. The inner product is taken to be $h^{ab} = \delta^{ab}$ while the structure constants are [1]¹

$$f^{abcd} = \frac{2\pi}{k}\epsilon^{abcd}.\quad (7)$$

In fact, as shown in [5, 8], for this choice of structure constants the 3-algebra theory is not as exotic as it first appears, for it reduces to a familiar Chern-Simons theory with gauge fields in the Lie algebra $su(2) + su(2)$ and matter in the bi-fundamental representation. The requirement that the theory is invariant under large gauge transformations imposes the usual quantization on the Chern-Simons coefficient which simply reads

$$k \in \mathbf{Z}.\quad (8)$$

This differs from the result quoted in [5] which, with our normalization, was $k \in 2\mathbf{Z}$. The correct normalization in the $SO(4)$ case can be seen by rewriting the action in terms of $SU(2) \times SU(2)$ gauge fields and, correcting a small typo in [8], noting that the coefficient of the CS term is $k/4\pi$. In the rest of this note we study a few elementary aspects of this $SO(4)$ theory.

¹We have redefined k by a factor of 2 relative to version 1 of this paper: $k_{\text{old}} = \frac{1}{2}k_{\text{new}}$. This is so that that the $k_{\text{old}} = \frac{1}{2}$ moduli space, mentioned only briefly in a footnote in v1, is elevated to the $k_{\text{new}} = 1$ moduli space, as befits the extended discussion given later in the paper. This redefinition is responsible for the apparent differences in subsequent formulae between v1 and the current version.

The Classical Moduli Space

We start by examining the vacuum moduli space of the classical theory, defined as solutions to $V(X) = 0$ modulo gauge transformations. This was previously discussed in [5, 8]. However, in both analyses, there was no obvious interpretation of the moduli space in terms of known M-theoretic objects. Here we clarify some points about the appearance of the dual photon which results in a simple M2-brane interpretation.

By a suitable gauge transformation, solutions to $V(X) = 0$ may be written as [5]

$$X^I = r_1^I T^1 + r_2^I T^2, \quad i.e. \quad X^I = \begin{pmatrix} r_1^I \\ r_2^I \\ 0 \\ 0 \end{pmatrix}. \quad (9)$$

However, as stressed in [8], there are additional gauge symmetries which preserve the form of X^I but act non-trivially on the two eight-dimensional vectors r_1 and r_2 . Since X transforms in the fundamental representation of $SO(4)$, we may act by $g \in SO(4)$ in the block diagonal form

$$g = \begin{pmatrix} g_1 & 0 \\ 0 & g_2 \end{pmatrix}, \quad (10)$$

where $g_1, g_2 \in O(2)$, with $\det g_1 = \det g_2$. Let us first look at a number of discrete symmetries. Since g_2 acts trivially on (9) we can effectively ignore it and simply look at $g_1 \in O(2, \mathbf{Z})$. There are three choices for g_1 which generate all of $O(2, \mathbf{Z})$ and act on r_1 and r_2 as

$$\begin{aligned} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} &: r_1 \rightarrow -r_1, \quad r_2 \rightarrow r_2 \\ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} &: r_1 \rightarrow r_1, \quad r_2 \rightarrow -r_2 \\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} &: r_1 \rightarrow r_2, \quad r_2 \rightarrow r_1. \end{aligned} \quad (11)$$

After imposing these discrete symmetries, r_1 and r_2 parameterize the 16-dimensional moduli space $\mathcal{M} \cong ((\mathbf{R}^8/\mathbf{Z}_2) \times (\mathbf{R}^8/\mathbf{Z}_2))/\mathbf{Z}_2$. However, we have still to divide out by the continuous $g_1 \in SO(2) \cong U(1)_{12}$ symmetry, which acts as

$$U(1)_{12}: z^I \rightarrow e^{i\theta} z^I \quad \text{where} \quad z^I = r_1^I + i r_2^I. \quad (12)$$

If we make use of all three discrete gauge symmetries (11), we already have the identification $z^I \rightarrow i z^I$. Thus, in order not to overreact, we must take the parameter θ to have range $\theta \in [0, \pi/2)$. Alternatively, we could impose just one discrete symmetry, say the last one which reads $z \rightarrow i \bar{z}$, and take $\theta \in [0, \pi]$.

Dividing out by this continuous gauge symmetry would seem to leave us with a 15-dimensional moduli space. This is a rather odd state of affairs and would contradict the expectations of supersymmetry. We will now show that by considering the unbroken gauge symmetry of the theory we will recover this lost dimension of moduli space. To see this, we proceed by writing down the low-energy effective action.

Because of the ϵ^{abcd} appearing in the covariant derivative (5), the $U(1)_{12}$ gauge symmetry (12) is associated to the gauge field A_μ^{34} . Normalizing so that z^I has charge +1, we define

$$B_\mu = \frac{4\pi}{k} A_\mu^{34}. \quad (13)$$

Then the kinetic terms on moduli space are given by

$$\mathcal{L}_{\text{moduli}} = -\frac{1}{2} |\mathcal{D}_\mu z^I|^2. \quad (14)$$

with $\mathcal{D}z = \partial z + iBz$. At a generic point in moduli space, there is also an unbroken $SO(2)$ symmetry [8], arising from the action g_2 in (10). We will call this symmetry $U(1)_{34}$. It is associated to the gauge field

$$C_\mu = \frac{4\pi}{k} A_\mu^{12}, \quad (15)$$

where the normalization is again taken to ensure that charged fields have charge ± 1 under C_μ . (Of course, by definition the moduli z^I themselves have charge zero under the unbroken symmetry). A mixed Chern-Simons term couples the B and C gauge fields;

$$\mathcal{L}_{cs} = \frac{k}{2\pi} \epsilon^{\mu\nu\lambda} B_\mu \partial_\nu C_\lambda. \quad (16)$$

It was shown in [6] that integrating out the broken gauge field B induces a Maxwell term for C , promoting it to a dynamical field. (In fact, the calculation in [6] was done at a non-generic point in moduli space with an unbroken $SU(2)$ gauge symmetry, but it proceeds in the same manner at a generic point). Here we instead replace the unbroken gauge field C with its dual photon, introduced in its usual guise as a Lagrange multiplier to impose the Bianchi identity on the field strength $G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$.

$$\mathcal{L}_{\text{dual}} = -\frac{1}{8\pi} \sigma \epsilon^{\mu\nu\lambda} \partial_\mu G_{\nu\lambda}. \quad (17)$$

The normalization is chosen such that $\sigma \in [0, 2\pi)$. To see this, note that $U(1)_{34} \subset SU(2)_{\text{diag}} \subset SO(4)$, with all matter fields in our theory living in the adjoint of

$SU(2)_{\text{diag}}$. The magnetic configurations of the theory are therefore given by the familiar Euclidean 't Hooft-Polyakov monopole solutions which satisfy the quantization condition,

$$\frac{1}{8\pi} \int d^3x \epsilon^{\mu\nu\lambda} \partial_\mu G_{\nu\lambda} \in \mathbf{Z}, \quad (18)$$

In the presence of the mixed Chern-Simons term (16), the shift symmetry of the dual photon becomes gauged under $U(1)_{12}$. This follows because the topological current $*G$, which generates the shift symmetry of the dual photon, is coupled to B_μ . It is also simple to see by collecting together the various pieces of the Lagrangian, which can be found in (14), (16) and (17),

$$\mathcal{L}_{\text{moduli}} + \mathcal{L}_{CS} + \mathcal{L}_{\text{dual}} = -\frac{1}{2} |\mathcal{D}_\mu z^I|^2 + \frac{1}{8\pi} \epsilon^{\mu\nu\lambda} (2kB_\mu + \partial_\mu \sigma) G_{\nu\lambda}. \quad (19)$$

This is invariant under the gauge action

$$U(1)_{12} : \quad z^I \rightarrow e^{i\theta} z^I \quad , \quad \sigma \rightarrow \sigma + 2k\theta \quad , \quad B_\mu \rightarrow B_\mu - \partial_\mu \theta. \quad (20)$$

Together with the discrete gauge symmetries (11), which now also induce a sign flip for σ .

In the next section, we will use (19) to analyze the moduli space dynamics. However, we can go further and eliminate the field strength $G_{\mu\nu}$. Since it is now unconstrained by the Bianchi identity, it acts as a Lagrange multiplier imposing the requirement that $B_\mu = -(1/2k)\partial_\mu \sigma$ is pure gauge. This results in the action

$$\mathcal{L} = -\frac{1}{2} |\partial_\mu z^I|^2 - \frac{i}{2k} z^I \partial_\mu \sigma|^2, \quad (21)$$

and we observe that σ can be eliminated by the field redefinition $z^I \rightarrow e^{-i\sigma/2k} z^I$. However, this transformation still leaves us with a number of discrete identifications which we now examine more carefully.

The Theory at Level $k = 1$ and $k = 2$

Let us return to the action in the form (19). For $k = 1$, we impose just one of the discrete symmetries, which we take to be $z \rightarrow z\bar{z}$, with $\theta \in [0, \pi]$. We can now fix the $U(1)_{12}$ gauge symmetry by imposing $\sigma = 0$, leaving us with remnant \mathbf{Z}_2 which acts by $\sigma \rightarrow \sigma + 2\pi$ and $z \rightarrow -z$. The moduli space at level $k = 1$ is thus,

$$\mathcal{M}_{k=1} \cong \frac{\mathbf{R}^8 \times \mathbf{R}^8}{\mathbf{Z}_2 \times \mathbf{Z}_2} \quad (22)$$

Writing $z = r_1 + ir_2$, the two \mathbf{Z}_2 factors act as $(r_1, r_2) \rightarrow (-r_1, -r_2)$ and $(r_1, r_2) \rightarrow (r_2, r_1)$. As observed in [13], this coincides with the infra-red limit of the moduli space of $d = 2 + 1$ dimensional, maximally supersymmetric Yang-Mills (SYM) with $SO(4)$ gauge group.

For $k = 2$, we may again fix the $U(1)_{12}$ gauge symmetry by setting $\sigma = 0$. Imposing all three discrete symmetries, we have $\theta \in [0, \pi/2)$ which now leaves no further residual transformation. The moduli space dynamics is simply given by the 8 complex scalars z^I , endowed with a flat metric and subject to the discrete symmetries (11). We conclude that the classical vacuum moduli space of the theory at level $k = 2$ is

$$\mathcal{M}_{k=2} \cong \frac{(\mathbf{R}^8/\mathbf{Z}_2) \times (\mathbf{R}^8/\mathbf{Z}_2)}{\mathbf{Z}_2}. \quad (23)$$

This coincides with the moduli space of $SO(5)$ SYM in the infra-red limit or, alternatively, the configuration space of two M2-branes in the background of the orbifold $\mathbf{R}^8/\mathbf{Z}_2$.

We strike a note of caution: the $k = 1$ and $k = 2$ theories are strongly coupled at all points in their moduli space. Nonetheless, we will assume that we can take (22) and (23) at face value. We take this as evidence that the $k = 1$ and $k = 2$ theories describe the infra-red fixed point of $SO(4)$ and $SO(5)$ SYM respectively². As we now review, in each case this can be understood as M2-branes moving in the orbifold background $\mathbf{R}^8/\mathbf{Z}_2$.

Let us briefly review a few pertinent facts about the M-theory orbifold $\mathbf{R}^8/\mathbf{Z}_2$. There are actually two different such orbifolds, distinguished by discrete torsion for G_4 arising because $H^4(\mathbf{RP}^7, \mathbf{Z}) \cong \mathbf{Z}_2$ [10]. The orbifolds with and without torsion are referred to as type-B and type-A respectively. The low-energy dynamics of N M2-branes in these orbifold backgrounds is thought to be governed by a maximally supersymmetric, $SO(8)$ invariant conformal fixed point. These arise as the strong coupling limit of maximally supersymmetric Yang-Mills (SYM) in $d = 2 + 1$ dimensions with gauge groups $O(2N)$, $SO(2N + 1)$, and $Sp(N)$. As explained in [10, 11], the fact that these three classical groups flow to one of only two possible theories implies non-trivial IR dualities between distinct UV theories. The RG flows occur as follows: $O(2N)$ SYM flows to the theory on M2-branes on the A-type orbifold; $SO(2N + 1)$ SYM flows to the theory on the

²This interpretation differs from that offered in [5, 6]. In particular, in [5], r_1 and r_2 were viewed as the relative separation of 3 M2-branes. However, neither the discrete symmetries, nor the flat diagonal metric, lend support to this.

B-type orbifold; while $Sp(N)$ SYM flows to either the theory on the A-type or B-type orbifold, depending on the expectation value of the dual photon. Comparing to our previous analysis, we see that the $k = 1$ theory describes two membranes on the A-type orbifold, while the $k = 2$ theory describes two membranes on the B-type orbifold.

The identification of the M2-brane Lagrangian (3) with M2-branes on an orbifold also resolves a puzzle raised in [8] regarding chiral primary operators. The bosonic, gauge invariant, operators of (3) live in tensor representations of $SO(8)$ with an even number of indices. Yet the chiral primary operators derived from M-theory on $AdS_4 \times \mathbf{S}^7$ live in the symmetric traceless s -index representations of $SO(8)$, with both even and odd s . However, pleasingly only the even s representations survive the orbifold projection in supergravity [12]. Although the AdS/CFT analysis is valid only at large N , it is comforting that this basic feature agrees with the $N = 2$ M2-brane theory.

The Theory at Level $k > 2$

Perhaps the most intriguing consequence of the Lagrangian (3) is the existence of a weakly coupled limit when $k \gg 1$. Understanding how such a limit arises from an M-theoretic description may be our best hope of getting a handle on the underlying microscopic degrees of freedom.

For $k > 2$, setting $\sigma = 0$ does not completely fix the $U(1)_{12}$ gauge action (20). There exists a residual \mathbf{Z}_k symmetry which leaves $\sigma = 0 \bmod 2\pi$ and is generated by,

$$z^I \rightarrow e^{i\pi/k} z^I. \quad (24)$$

As pointed out in [13], this \mathbf{Z}_k action does not commute with the \mathbf{Z}_2 actions of equation (11). Between them they generate the dihedral group D_{2k} . We conclude that the moduli space is given by,

$$\mathcal{M}_k \cong \frac{\mathbf{R}^8 \times \mathbf{R}^8}{D_{2k}} \quad (25)$$

However, while the group D_{2k} has a simple action on the moduli space, it does not appear to have a such a description on the spacetime transverse to the M2-branes for $k > 2$. In particular, it does not leave the distances between branes fixed. Needless to say, it would be potentially rather interesting to better understand the microscopic meaning of this quotient action and these higher k theories. A curious observation of [13] is that the moduli space for $k = 3$ coincides with the infra-red limit of SYM with G_2 gauge group.

The Spectrum and Non-Abelian Gauge Restoration

We note that the \mathbf{Z}_k action (24) would not make much sense on a pair of D-branes. One simple way to see this is to note that it does not preserve the distance between the two branes. In string theory this distance dictates the spectrum of massive states arising from stretched strings. Yet the M2-brane theory appears to be blind to the transverse distance between the two branes. It knows only about transverse areas! This is clear if we look at the classical mass spectrum, which we trust for $k \gg 1$. Sitting at a generic point in moduli space, we may employ the $SO(8)$ R-symmetry to rotate the M2-branes to lie in the $X^7 - X^8$ plane. Then the mass of states is given by,

$$M = \frac{4\pi}{k} A \quad (26)$$

where $A = \frac{1}{2}|r_1^7 r_2^8 - r_1^8 r_2^7| = \frac{1}{4}|\bar{z}^7 z^8 - \bar{z}^8 z^7|$ is the area of the triangle formed by the two M2-branes and the orbifold fixed point. This is manifestly invariant under the \mathbf{Z}_k action.

We finish with a few comments on the implications of this mass formula. Firstly, it implies that new states become massless when the branes become co-linear with the orbifold fixed point. This is to be contrasted with the familiar statement that states on D-branes become massless when branes coincide. Let us see how these massless states arise. In generic vacua the R-symmetry is broken to $SO(6)$ and, as we noted previously, a $U(1)_{34}$ gauge symmetry survives. However, when the branes are co-linear, and the R-symmetry is broken to $SO(7)$, a full $SO(3)$ gauge symmetry is left unbroken. This was the situation examined in [6] where it was shown that, upon integrating out the broken gauge generators, this $SO(3)$ gauge field becomes dynamical. These are the new massless states.

The emergence of this dynamical $SO(3)$ gauge field is something of a blessing, for it removes a potential difficulty in interpreting the expectation value (9) as the position of two branes. The problem is that whenever the branes are co-linear, one can change the relative positions of the branes through a gauge transformation. For example, the $SO(7)$ preserving expectation values

$$X^I = r^I(c_1 T^1 + c_2 T^2) \quad (27)$$

are gauge equivalent for all c_1 and c_2 such that $c_1^2 + c_2^2$ is constant. Naively this would equate configurations with different separations between co-linear branes and the fixed point. In fact the theory does distinguish between these configurations, but it is somewhat hard to see explicitly. The presence of the dynamical, unbroken,

$SO(3)$ gauge field means that there is a non-Abelian dual photon, whose expectation value will determine the relative positions of the branes. This is entirely analogous to the situation of two D2-branes in IIA string theory, for which the moduli space is $(\mathbf{R}^7 \times \mathbf{S}^1)/\mathbf{Z}_2$. Even at the origin of \mathbf{R}^7 , where the gauge group is unbroken, the branes may still be separated in a non-singular fashion along the M-theory circle. However, seeing this how this explicitly arises from the non-Abelian dual photon is difficult.

A related fact is that the appearance of the massless states when the branes are co-linear does not necessarily imply a singularity in the low-energy effective theory. This is exemplified in the D2-brane, where there are only isolated singularities in the moduli space, rather than a whole \mathbf{S}^1 's worth of singularities at the origin of \mathbf{R}^7 . Indeed, from the M-theory perspective, the generic point with co-linear branes should be smooth. More precisely, we expect that, in the vacua (27), there is just a single singularity for the $k = 1$ theory, corresponding to the two two branes sitting on top of each other. For the $k = 2$ theory, there should be two singularities, the first corresponding to the two branes sitting on top of each other, while the second corresponds to one brane sitting on the orbifold fixed plane which is now expected to result in a non-trivial fixed point.

Finally, it is tempting to believe that the mass formula (26) is hinting at some fundamental degree of freedom of M-theory. The fact that the mass should scale as an area is, for $k \gg 1$, a consequence of conformal invariance, and the triangle is the only natural area in the theory. Nonetheless, the appearance of such a “3-pronged” object is intriguing, not least because such states would naively explain the famous N^3 entropy of the M5-brane theory [14]. However, quite how one could scale such states to account for the $N^{3/2}$ entropy for M2-branes, in a controllable weakly coupled regime, appears as tantalisingly mysterious as ever.

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